Education

KwaZulu-Natal Department of Education

MATHEMATICS P2

MARKING GUIDELINE

PREPARATORY EXAMINATION

SEPTEMBER 2018

NATIONAL SENIOR CERTIFICATE

GRADE 12

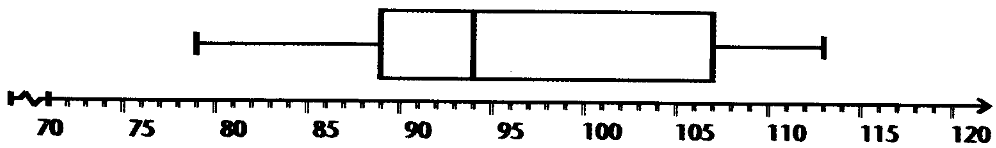
MARKS: 150

This marking guideline consists of 14 pages.

QUESTION 1

1.1	strong positive trend	✓ A strong positive	(1)
1.2	(38; 127)	✓ A answer	(1)
1.3	$a = 68{,}66$ $b = 2{,}46$ $y = 68{,}66x + 2{,}46x$	✓ A a = 68,66 ✓ A b = 2,46 ✓ CA equation	(3)
1.4	$y = 68{,}66 + 2{,}46\,(24)$ $= 127{,}7$ $= 127$	✓ CA ✓ CA answer	(2)
			[7]

QUESTION 2

2.1	Mean weight $= \bar{x} = \frac{1443}{15}$ $= 96{,}2\ kg$	✓ A sum divided by 15 ✓ CA answer (only if dividing by 15)	(2)
2.2	σ = standard deviation = 11,27	✓✓AA answer	(2)
2.3	$(\bar{x} - \sigma ;\ \bar{x} + \sigma)$ = (84,93 ; 107,47) Therefore 2 scores are less than the standard deviation	✓CA identify range ✓CA answer	(2)
2.4		✓ A min value 79 ✓ A $Q_1 = 89$ ✓ A $Q_2 = 94$ ✓ A Q3 = 107 ✓ A max value = 113	(5)
2.5	IQR = $Q_3 - Q_1$ = 107 − 89 = 18	✓ CA difference ✓CA answer	(2)
2.6	$\bar{x}$ − median = 96,2 − 94,00 = 2,2 Data is positively skewed.	✓ CA answer	(1)
			[14]

QUESTION 3

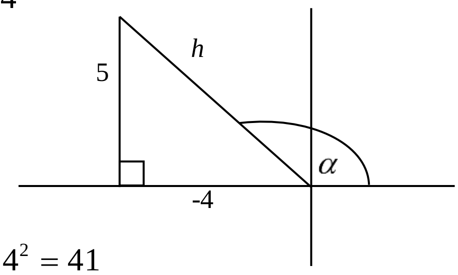
3.1.1	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-5}{4-(-4)}$ $= \frac{-4}{8}$ $= \frac{-1}{2}$	✓ A substitution into gradient formula ✓CA answer (provided – answer)	(2)
3.1.2	$y = mx + c$ $5 = -\frac{1}{2}(-4) + c$ $c = 3$ $y = -\frac{1}{2}x + 3$	✓ CA substituting point and gradient of line AB ✓CA answer	(2)
3.1.3	$m_{CD} = 2$ CD $\perp$ AB $y = mx + c$ $-4 = 2(-1) + c$ $c = -2$ $y = 2x - 2$	✓ CA CD $\perp$ AB ✓ A substituting point (-1;-4) ✓ CA answer	(3)
3.1.4	$\therefore\ 2x - 2 = -\frac{1}{2}x + 3$ $\frac{5}{2}x = 5$ $\therefore\ x = 2$ $\therefore\ y = 2(2) - 2$ $= 2$ $\therefore$ E(2; 2)	✓ CA Equating ✓ CA $x = 2$ ✓CA $y = 2$ (CA if both co-ordinates are positive)	(3)

3.1.5	$m_{CB} = \frac{1-(-4)}{4-(-1)}$ $=1$ Equation of line passing through A parallel to BC $=1$ $y = mx + c$ $5 = 1(-4) + c$ $c = 9$ $y = x + 9$	✓ A substitution into gradient formula ✓CA gradient value ✓ CA gradient of Line parallel ✓ A substitution of point (- 4 ; 5) ✓ CA answer	(5)
3.2	$\tan\theta = 1$ $\theta = 45^\circ$	✓ CA $\tan\theta = 1$ ✓ CA answer	(2)
3.3	CE = $\sqrt{(2-(-1)^2) + (2-(-4))^2}$ $= \sqrt{9+36}$ $= \sqrt{45}$ $= 3\sqrt{5}$ AE = $\sqrt{(2-(-4))^2 + (2-5)^2}$ $= \sqrt{36+9}$ $= \sqrt{45}$ $= 3\sqrt{5}$ Area of ΔAEC = $\frac{1}{2}$ base x height $= \frac{1}{2} . \ 3\sqrt{5} \ \text{x} \ 3\sqrt{5}$ $= \frac{1}{2} . \ 9 \text{ x } 5$ $= \frac{45}{2}$ $=$ 22,5 units^2	✓CA answer ✓ CA answer ✓ CA Correct substitution into Area formula ✓ CA Answer	(4) **[21]**

QUESTION 4

4.1	P(6 ; - 2)	✓A x – value ✓A y - value	(2)
4.2	$2x-4=0$ $x=2$ $S(2\,;0)$	✓ A equating to 0 ✓ A x – value	(2)
4.3	$A\hat{B}C=90^\circ$ Angle in a semi-circle $m_{BC}=-\frac{1}{2}$ AB $\perp$ BC $y=mx+c$ $2=-\frac{1}{2}(3)+c$ $c=\frac{7}{2}$ $y=-\frac{1}{2}x+\frac{7}{2}$	✓ A Statement ✓ A gradient of BC ✓ A substitution of point (3 ;2) ✓CA answer	(4)
4.4	R(7 ; 0) x int of BC $BR^2=(7-3)^2+(0-2)^2=20$ $(x-7)^2\{+(y-0)^2=20$	✓CA for 7✓A for 0 coordinates of R ✓CA subst. into distance formula ✓CA radius value ✓CA answer	(5)
4.5	$m_{PS}=-\frac{1}{2}$ $\therefore PS\,//\,CB$ equal gradients A(1; - 2) midpoint formula Since the y – coordinates of A and P is – 2 Therefore AC//SR OR m_{AC} = 0 … (both y values are the same) m_{SR} = 0 … (x–axis) $\therefore$ m_{AC} = m_{SR} $\therefore$ AC//SR	✓A✓A gradient of PS ✓A PS//CB ✓A coordinates of A ✓A Reasoning ✓ A Statement ✓A Reason ✓A Statement ✓ A Reason ✓ A m_{AC} = m_{SR}	**(5)** **[18]**

QUESTION 5

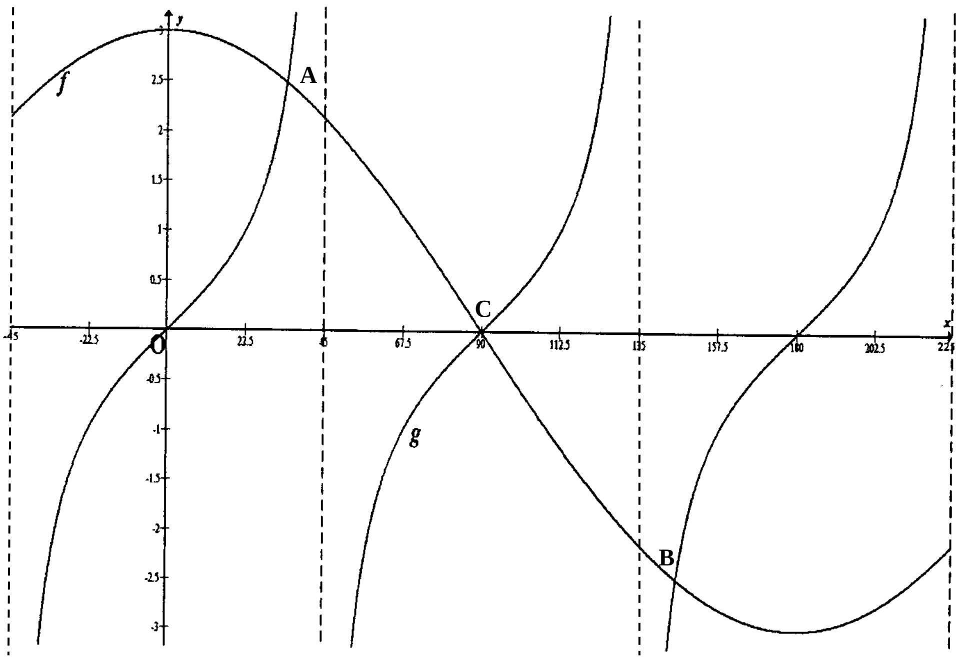
5.1	$4\tan\alpha + 5 = 0$ $\tan\alpha = \frac{-5}{4}$  $h^2 = 5^2 + 4^2 = 41$ $\therefore h = \sqrt{41}$ $\cos 180^o = -1$ $\sin(-150^o) = -\sin 30^o$ $= -\frac{1}{2}$ $\sqrt{41}\left(\frac{-4}{\sqrt{41}}\right) - 4\left(-\frac{1}{2}\right)(-1) = -4 - 2 = -6$	✓A diagram in the correct quadrant ✓ A $\sqrt{41}$ ✓ A -1 ✓ A $-\frac{1}{2}$ ✓CA answer	(5)

5.2.1	$\frac{\cos 99^o}{\cos 33^o} \; \frac{-\sin 99^o}{\sin 33^o}$ $= \frac{\cos 99^o \sin 33^o - \sin 99^o \cos 33^o}{\cos 33^o \sin 33}$ $\frac{-\left[\sin 99^o \cos 33^o - \cos 99^o \sin 33^o\right]}{\cos 33^o \sin 33^o}$ $= \frac{-\sin(99^o - 33^o)}{\cos 33^o \sin 33^o}$ $= \frac{-\sin 66^o}{\cos 33^o \sin 33^o}$ $= \frac{-2\sin 33^o \cos 33^o}{\cos 33^o \sin 33^o}$ $= -2$	✓ A Simplification ✓ A Taking negative sign out ✓ A sin (99° -33°) ✓ A sin 66° ✓ A 2 sin 33° cos 33° ✓ A answer	(6)
5.2.2	$= \frac{-\cos 40^\circ - (\cos\theta)}{\sin 50^\circ + \cos\theta}$ $= \frac{-\cos 40^o - (\cos\theta)}{\cos 40^o + \cos\theta} = \frac{-(\cos 40^\circ + \cos\theta)}{(\cos 40^\circ + \cos\theta)}$ $= -1$	✓ A -cos 40° ✓A $\cos\theta$ (numerator) ✓ A sin 50° ✓ $\cos\theta$ (denominator) ✓ CA answer	(5)
5.3	$\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{\cos x}{\sin x}$ $LHS = \frac{2\sin^2 x}{\frac{2\sin x}{\cos x} - 2\sin x \cos x}$ $= \frac{2\sin^2 x}{\frac{2\sin x - 2\sin x \cos^2 x}{\cos x}}$ $= \frac{2\sin^2 x.\cos x}{2\sin x - 2\sin x \cos^2 x}$ $= \frac{2\sin^2 x \cos x}{2\sin x\left[1 - \cos^2 x\right]}$ $= \frac{2\sin x \cos x}{\sin^2 x}$ $= \frac{\cos x}{\sin x}$ $= RHS$	✓ A $2\sin x \cos x$ ✓ A $\frac{\sin x}{\cos x}$ ✓ A Simplification ✓ A removal of common factor of 2 sin x ✓ A $\frac{\sin x \cos x}{\sin^2 x}$	(5)

5.4	$8\sin\theta\cos\theta = -2\sqrt{3}$ $\frac{8\sin\theta\cos\theta}{4} = \frac{-2\sqrt{3}}{4}$ $2sin\theta cos\theta = \frac{-\sqrt{3}}{2}$ $Sin2\theta = \frac{-\sqrt{3}}{2}$ reference angle = 60° $2\theta = (180° + 60°) + k.360°, k \in Z$ $2\theta = 240° + k.360°, k \in Z$ $\theta = 120° + k.180°, k \in Z$ OR $2\theta = (360^o - 60^o) + k\ .\ 360^o,\ k \in Z$ $2\theta = 300^o + k\ .\ 360^o,\ k \in Z$ $\theta = 150^o + k.\ 180^o,\ k \in Z$	✓ A dividing by 4 both sides ✓ A $2\sin\theta\cos\theta = \sin 2\theta$ ✓ A 60^o ✓ CA 240^o ✓ CA $\theta = 120° + k.180°, k \in Z$ ✓ CA 300^o ✓ CA $\theta = 150^o + k\ .\ 180^o,\ k \in Z$	(7)
			[29]

QUESTION 6

6.1



✓A shape of *f*

✓ A shape of *g*

✓✓A A asymptotes

✓A *x*-intercepts of *f*

✓ A Turning points of *f*

✓ A *x*-intercepts of *g*

✓ A 3 intersection points

(8)

6.2	the graphs intersect at A, B and C. At A we have $x = 34^o$, at C we have $x = 90^o$ and by using symmetry we get at B, $x = 180^o - 34^o = 146^o$.	✓ A using symmetry ✓A answer Answer only full marks	(2) **[10]**

QUESTION 7 As a result of the typographical error in the question paper this question will not be marked – Total of paper will now be 144 marks but must be converted to 150 for recording purposes)

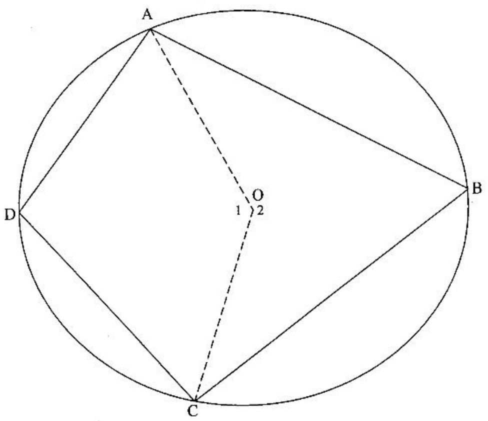
7.	In Δ PQS $\tan y = \dfrac{h}{PQ}$ $\therefore PQ = \dfrac{h}{\tan y}$ $= \dfrac{h\cos y}{\sin y}$ In Δ PQR $P\hat{Q}R = \dfrac{180^o - 2y}{2}$ $= 90^o - y$ $\therefore \dfrac{PR}{\sin(90^o - y)} = \dfrac{PQ}{\sin 2y}$ $\therefore PR = \dfrac{PQ\cos y}{\sin 2y}$ $= \dfrac{h\cos y}{\sin y} \cdot \dfrac{\cos y}{\sin 2y}$ $= \dfrac{h\cos^2 y}{\sin y \,.\, \sin 2y}$	✓ $\tan y = \dfrac{h}{PQ}$ ✓ $PQ = \dfrac{h\cos y}{\sin y}$ ✓ $P\hat{Q}R = 90^o - 2y$ ✓ applying sine rule ✓ $\sin (90^o - y) = \cos y$ ✓ subt $PQ = \dfrac{h\cos y}{\sin y}$	**[6]**

QUESTION 8

8.1	BC = 15 cm line from centre ⊥ chord	✓✓A A S & R	(2)
8.2	OC = $2a$	✓A answer	(1)
8.3	OB = $3a$	✓CA answer	(1)
8.4	$\therefore (3a)^2 = (2a)^2 + (15)^2$ (Pythagoras) $\therefore 9a^2 = 4a^2 + 225$ $\therefore 5a^2 = 225$ $\therefore a^2 = 45$ $\therefore a = \sqrt{45}$ $AB^2 = 15^2 + (5a)^2$ (Pythagoras) $= 225 + 25\ (45)$ $\therefore AB = \sqrt{1350} = 15\sqrt{6}$	✓CA applying Pythagoras ✓CA $a = \sqrt{45}$	

	$= 36{,}7$ cm	✓CA answer	(3)
8.5	$A\hat{C}B = 90^\circ$ $\therefore$ AB is a diameter of circle CAB [converse of angle in semi circle] $\therefore$ Radius $= \frac{1}{2}$ diameter $= \frac{1}{2}$ 36,7 cm $= 18{,}4$ cm	✓A Reason ✓CA answer	(2) **[9]**

QUESTION 9

9.1	 Construction: Draw AO and CO <u>Proof:</u> $\hat{O}_1 = 2\hat{B}$ … $\angle$ at centre = 2 $\angle$ at circle $\hat{O}_2 = 2\hat{D}$ … $\angle$ at centre = 2 $\angle$ at circle $\hat{O}_1 + \hat{O}_2 = 360^o$ $2\hat{B} + 2\hat{D} = 360^o$ $\hat{B} + \hat{D} = 180^o$	✓ A Construction ✓ A S/R ✓ A S/R ✓ A $\hat{O}_1 + \hat{O}_2 = 360^o$ (revolution) ✓ A Substitute for $\hat{O}_1$ and $\hat{O}_2$ (All Accuracy Marks)	(5)
9.2.1	$\hat{K}_1 = x = \hat{K}_2$ … KM bisects $L\hat{K}N$ $\hat{O}_1 = 2x$ angles opp = sides $\therefore \hat{L} = x$ $\angle$ at centre = 2 $\angle$ at circumference $\therefore \hat{K}_1 = \hat{L} = x$ $\therefore$ TK = TL (sides opposite equal angles)	✓ S ✓ R ✓ S ✓ R ✓ R	(5)

9.2.2	$\hat{T}_1 = 2x$ ... ext $\angle$ of Δ QKL $\hat{T}_1 = \hat{O}_1 = 2x$ $\therefore$ KOTP is a cyclic quadrilateral …converse of $\angle$'s on the same segment equal.	✓ S ✓R ✓ R (All Accuracy Marks)	(3)
9.2.3	$\hat{P}_1 = L\hat{K}N$ ... Angles in the same segment $= 2x$ $\therefore \hat{P}_1 = \hat{T} = 2x$ $\therefore$ PN // MK ... alt $\angle$'s proved equal	✓ S ✓R ✓ R (All Accuracy Marks)	(3)
			[16]

QUESTION 10

10.1	$\frac{AS}{SP} = \frac{AR}{RB}$ ... RS // BP $= \frac{3}{2}$ $\therefore \frac{AS}{SC} = \frac{3}{7}$	✓ S/R ✓ $\frac{3}{2}$ ✓ $\frac{3}{7}$ (All Accuracy Marks)	(3)
10.2	$\frac{RT}{TC} = \frac{SP}{PC}$ … RS//TP $= \frac{2}{5}$	✓ S/R ✓ $\frac{2}{5}$	(2)
10.3	$\frac{\Delta ARS}{\Delta ABC} = \frac{\Delta ARS}{\Delta ARC} \times \frac{\Delta ARC}{\Delta ABC}$ $= \frac{3}{10} \times \frac{3}{5}$ $= \frac{9}{50}$	✓ratio of Δ's ✓substitution ✓CA answer (All Accuracy Marks if not indicated)	(3) **[8]**

QUESTION 11

11.1	In Δ PAT and Δ PCA 1. $\hat{P}$ *is common* 2. $\hat{A}_1 = \hat{C}_1$ tan chord thrm. 3 $P\hat{T}A = P\hat{A}C$ sum of angles in triangle $\therefore$ Δ PAT /// ΔPCA ($(\angle\angle\angle)$ $\therefore \frac{PA}{PC} = \frac{PT}{PA}$ (/// Δ's) $\therefore PA^2 = PC \cdot PT$	✓ S (identifying triangles) ✓ S ✓ S ✓ S/R ✓ S All accuracy marks	(5)
11.2	$PA^2 = PC \cdot PT$ $\therefore 36 = (x + 5)\ x$ $\therefore\ 36 = x^2 + 5x$ $\therefore\ x^2 + 5x - 36 = 0$	✓ A subst. ✓ A simplifying	(2)
11.3	$(x + 9)(x - 4) = 0$ $x = -9$ or $x = 4$ N/A $\therefore$ PT = 4 units	✓ A factorising ✓ A PT = 4	(2)
11.4	$\frac{PD}{PA} = \frac{PT}{PC}$ (AC//DB; prop. theorem) $DP = \frac{4}{9} \cdot 6$ $= \frac{8}{3}$	✓S ✓R ✓CA answer	(3) **[12]**

TOTAL MARKS: 150